SEAHAM HIGH SCHOOL

Calculation Framework

Aim of Calculation Policy

Seaham High School is committed to raising standards of numeracy so that all students can develop the ability to use numeracy skills effectively in all areas of the curriculum and can confidently transfer skills across subject areas.

Being numerate requires a confidence and competency in applying mathematical skills in everyday life. To this end, all teachers have a responsibility for promoting the use of numeracy in their subjects. Students often find it difficult to transfer the skills learned in the maths classroom to other subjects and vice versa.

The aim of this Calculation Framework is to create a consistent approach that will be adopted by all teachers so that students are taught topics in the same manner, with the same techniques, across all subject areas. This will allow students to see the links between maths and other subjects, supporting and making it easier for them to transfer their skills.

It should be noted that if a student already has an alternative method or is struggling with the method agreed in this framework, the use of an alternative method to aid their understanding is entirely justifiable.

Contents

Number

Algebra

Geometry

Data

Number

Place Value

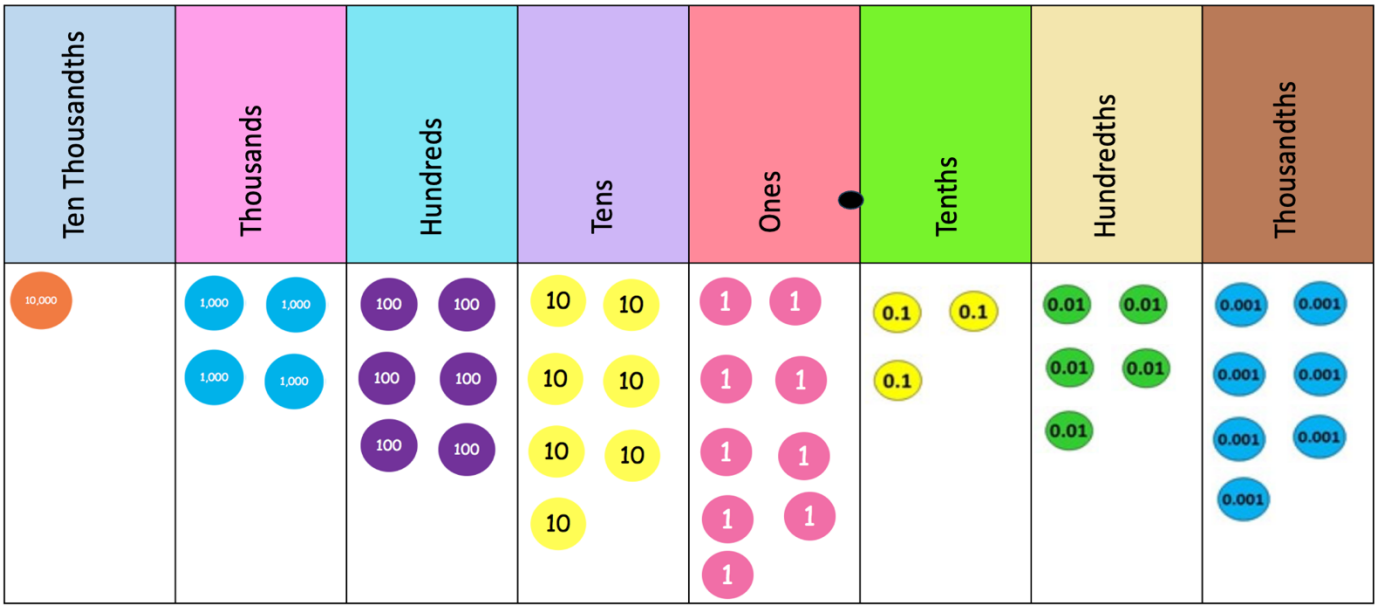
Ten Millions	Millions	Hundred Thousandths	Ten Thousandths	Thousands	Hundreds	Tens	Ones	Tenths	Hundredths	Thousandths
								$\frac{1}{10}$	$\frac{1}{100}$	$\frac{1}{1000}$
10,000,000	1,000,000	100,000	10,000	1000	100	10	1	0.1	0.01	0.001
10^7	10^6	10^5	10^4	10^3	10^2	10^1	10^0	10^{-1}	10^{-2}	10^{-3}

7 6 4 5 . 3 8 2

Can be written as:
- 6 hundreds
- 600
- 6×10^2

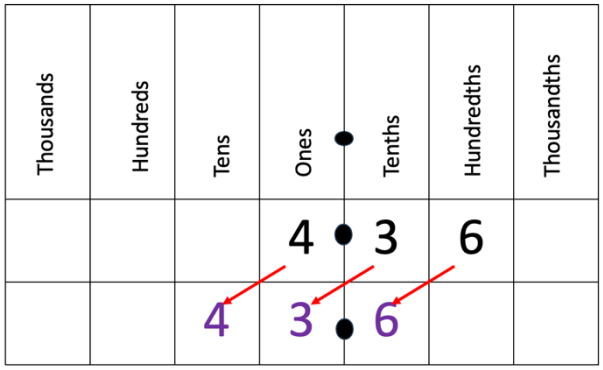
(Worth 60 tens)

Can be written as:
- 3 tenths
- 0.3
- $\frac{3}{10}$

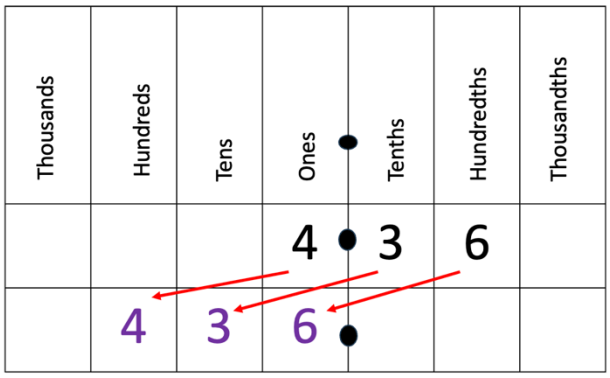


Place value counters will be used to support understanding.

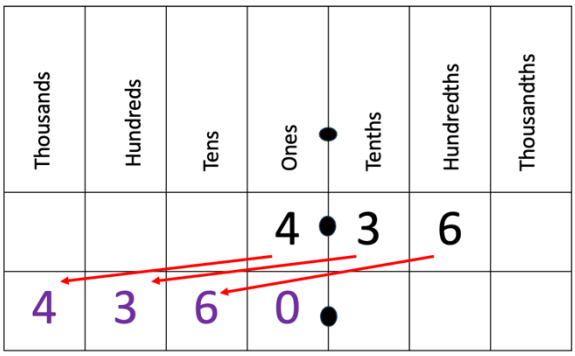
Multiplying by 10/100/1000



To multiply by 10, move the digits one place left.
64.3 x 10 = 43.6

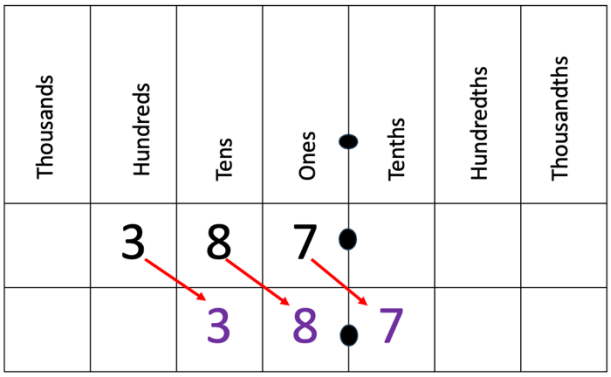


To multiply by 100, move the digits two places left.
4.36 x 100 = 436

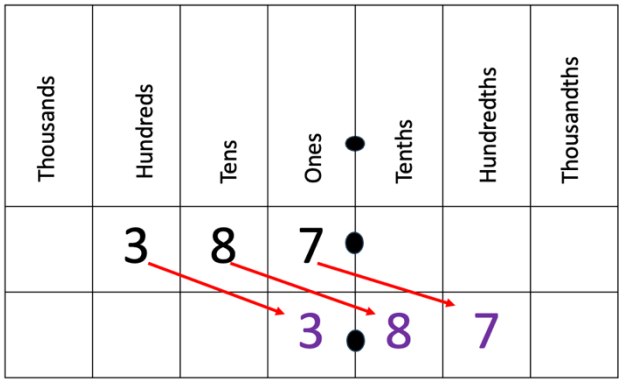


To multiply by 1000, move the digits three places left.
4.36 x 1000 = 4360

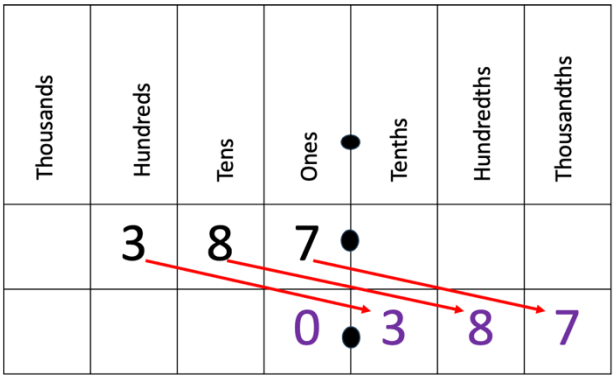
Dividing by 10/100/1000



To divide by 10, move the digits one place right.
387 ÷ 10 = 38.7



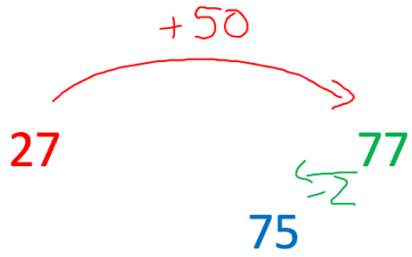
To divide by 100, move the digits two places right.
387 ÷ 100 = 3.87



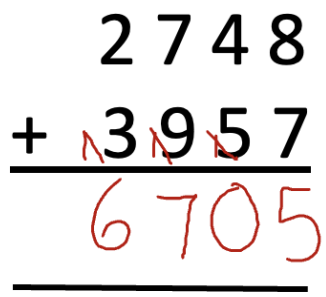
To divide by 1000, move the digits three places right.
387 ÷ 1000 = 0.387

Addition

Mental Methods

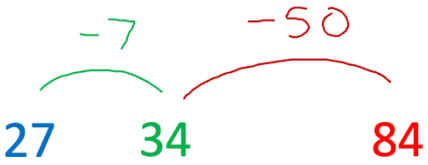
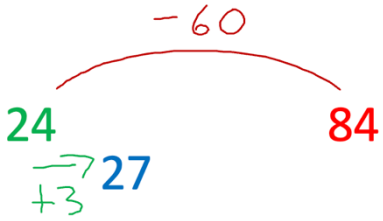
Method 1	Method 2
27 + 48 20 + 40 = 60 7 + 8 = 15 60 + 15 = 75 Partition each of the numbers into tens and ones.	27 + 48  Round up to the nearest ten and then add. 27 + 50 = 77 (2 too many) 77 - 2 = 75

Written Method

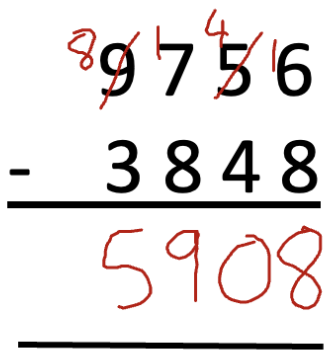
Method
 Column addition is used. Line up the numbers according to their place value. Always start addition on the right by adding the ones first. If the total is 10 or more, then numbers must be **regrouped.** In the example, 8 + 7 is regrouped into 1 ten and 5 ones. 7 + 9 hundreds would be regrouped into 1 thousand and 6 hundreds.

Subtraction

Mental Methods

Method 1	Method 2
84 - 57 	84 - 57 
Partition the second number into tens and ones.	Round up to the nearest ten and then subtract. 84 – 60 = 24 (3 too many) 24 + 3 = 27

Written Method

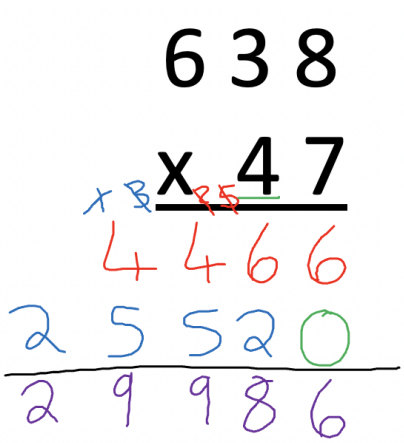
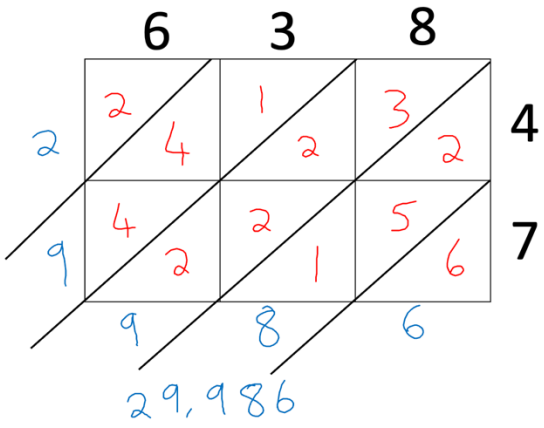
Method
 Column subtraction is used. Line up the numbers according to their place value. Always start on the right by subtracting the ones first. If the top number is lower than the bottom number than an **exchange** must take place. For example, 6 is lower than 8 so one ten is exchanged for 10 ones. 16 – 8 = 8 Note 4 tens are left after the exchange.

Multiplication

Mental Method

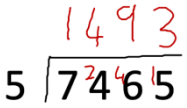
Method
38 x 7 30 x 7 = 210 8 x 7 = 56 210 + 56 = 266 Partition and then multiply by the tens and ones respectively. Add together each total.

Written Methods

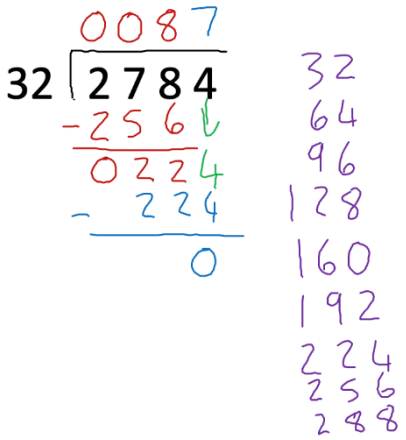
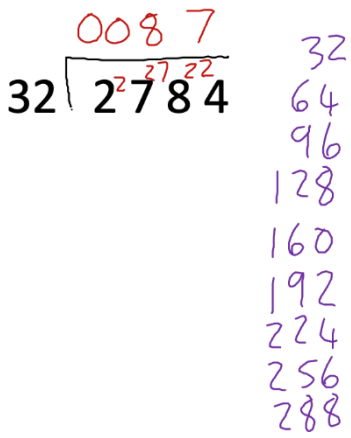
Method 1	Method 2
	
Multiply 638 by 7 (top red row). Multiply 638 by 40 (second blue row). Note the zero place value holder (in green) to make the second row ten times bigger because the multiplication is 40 not 4. Add the products together.	Draw a grid with diagonals. Write one number vertically and the other horizontally. Multiply the digits above and to the left of each square. Add down the diagonals.

Division

Short Division

Method
7465 ÷ 5 
Place the number to be divided (divisor) outside. There is one 5 in 7 with a remainder of 2 so the first digit of the answer is 1 and the 2 is placed with the 4 to become 24. There are four 5s in 24 with a remainder of 4. There are nine 5s in 46 with a remainder of 1. There are three 5s in 15 with no remainder. The answer (quotient) is 1493.

Long Division

Method 1	Method 2
2784 ÷ 32  1) List the multiples of 32. 2) 32 won't divide into 2 or 27 so needs to be divided into 278. 3) The closest multiple is 256 which is 8 lots of 32. Complete subtraction of 274 – 256 = 22. 4) Bring down the 4 to make 224. 5) 32 divides into 224 seven times. 6) Complete 224 – 224 to get 0. The answer is 87.	2784 ÷ 32  Always list the multiples of the number to be divided in (divisor) first. Use the same process as used for short division but with the two digit number.

Order of Operations

Method		
1st ()	Brackets	B
2nd x^2, x^3, √	Indices	I
3rd (equal priority)	Division Multiplication	D M
4th (equal priority)	Addition Subtraction	A S

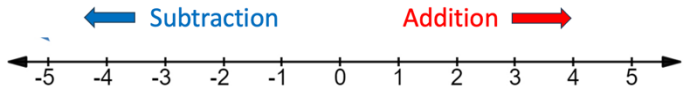
e.g. 50 – 4 x 5

Multiplication takes priority over subtraction so 4 x 5 is carried out first.

50 – 4 x 5
= 50 – 20
= 30

Directed Numbers

Addition and Subtraction

Method
 + + = + e.g. 5 + 6 = 11 + - = - e.g. 5 + (-6) = 5 – 6 = -1 - + = - e.g. 5 – 6 = -1 - - = - e.g. 5 – (-6) = 5 + 6 = 11

Counters can be used to support understanding.

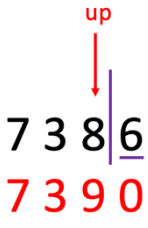
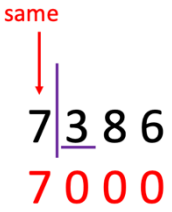
5 + (-3) = 2	
-3 + (-4) = -7	
-5 – (-2) = -3	

Multiplication and Division

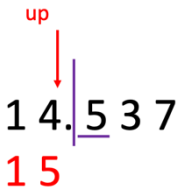
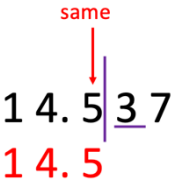
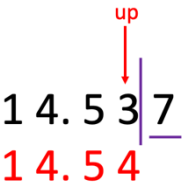
Method
Positive x positive = positive e.g. 6 x 4 = 24 Positive x negative = negative e.g. 6 x -4 = -24 Negative x positive = negative e.g. -6 x 4 = -24 Negative x negative = positive e.g. -6 x -4 = 24 The same rules apply for division.

Rounding and Estimation

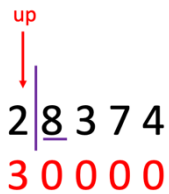
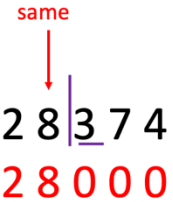
Rounding

Nearest 10	Nearest 100	Nearest 1000
		
Put a line after the place value being rounded to and underline the digit next to it. The underlined digit indicates whether the number should round up or stay the same. 0, 1, 2, 3, 4 -> stays the same 5, 6, 7, 8, 9 -> rounds up		

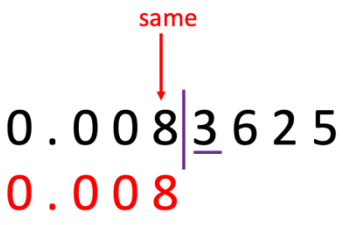
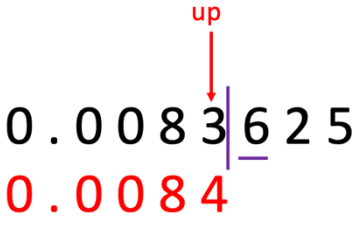
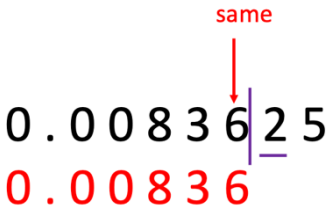
Rounding to Decimal Places

Nearest whole	1 d.p.	2.d.p
		

Rounding to Significant Figures

1 s.f.	2 s.f.	3 s.f.
		
Put a line after the place value being rounded to and underline the digit next to it. The underlined digit indicates whether the number should round up or stay the same. 0, 1, 2, 3, 4 -> stays the same 5, 6, 7, 8, 9 -> rounds up All numbers after the rounded digit then turn to zero up to and including the ones.		

Rounding to Significant Figures Decimals

1 s.f.	2 s.f.	3 s.f.
		

Zeros before a non-zero number are not significant.

Put a line after the place value being rounded to and underline the digit next to it. The underlined digit indicates whether the number should round up or stay the same.

0, 1, 2, 3, 4 -> stays the same
5, 6, 7, 8, 9 -> rounds up

Zeros after a non-zero number are significant so do not turn digits after the rounded digit to zeros.

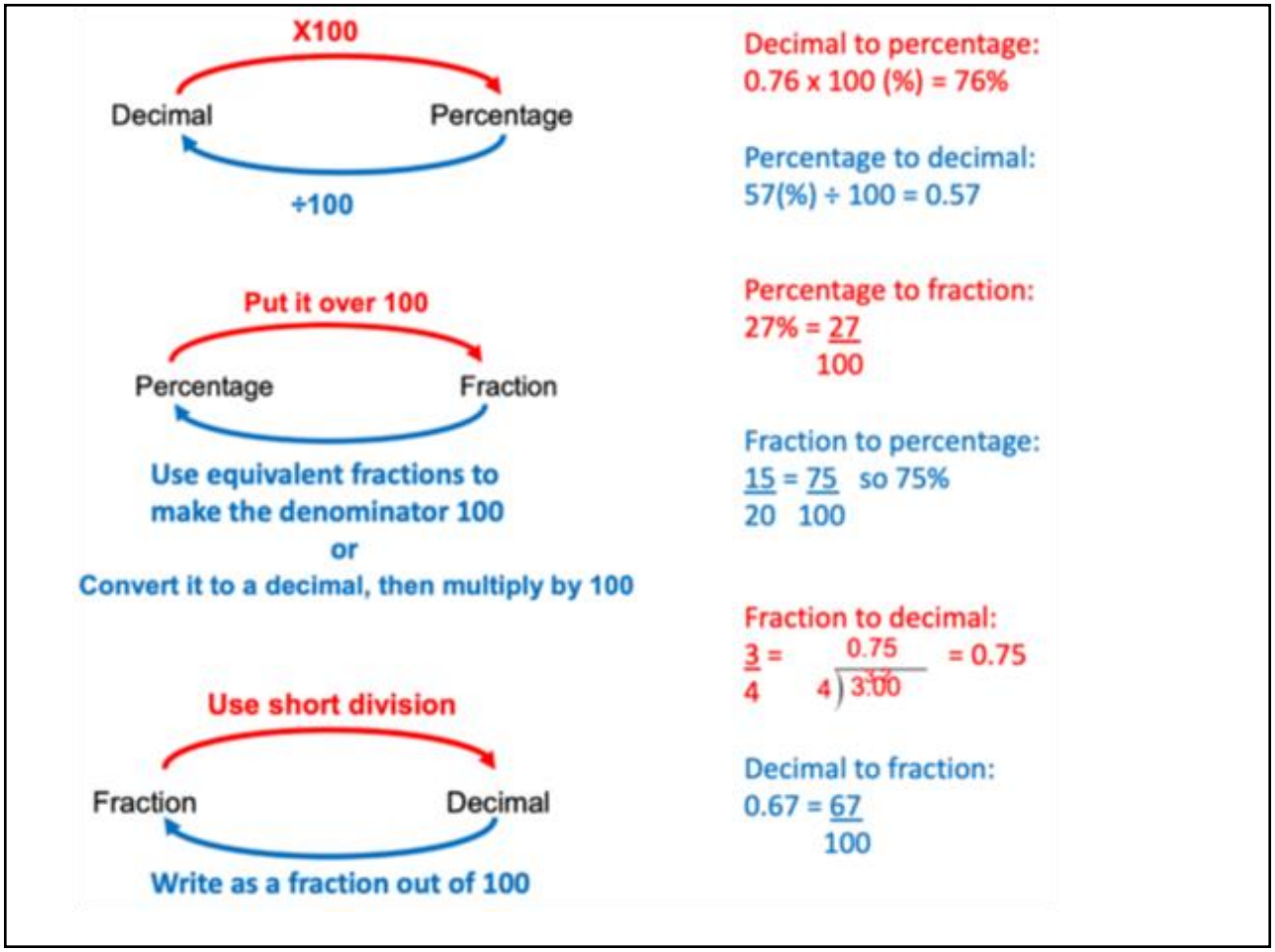
Estimation

Example 1	Example 2
Estimate 24.7×4.2. $= 20 \times 4$ $= 80$ Numbers should be rounded to one significant figure when estimating.	Estimate $\frac{801 \times 10.4}{0.49}$ $= \frac{800 \times 10}{0.5}$ $= \frac{8000}{0.5}$ (x10) $= \frac{80000}{5}$ (x10) $= 16000$ Tricky calculations can be resolved by making adjustments so that all numbers are whole numbers.

Number Properties

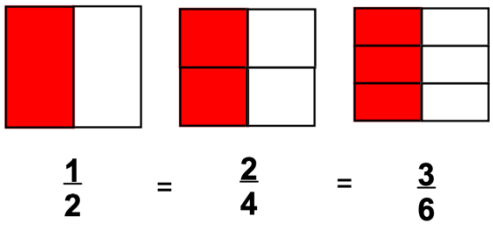
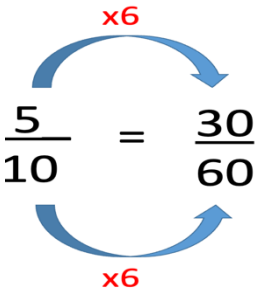
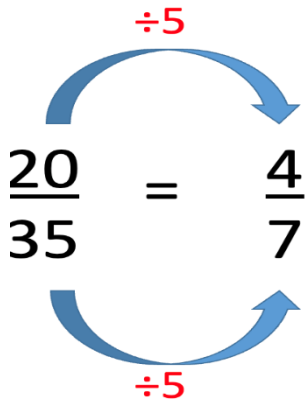
Type	Example	Definition
Even numbers	2, 4, 6, 8, 10	Numbers that are divisible by 2 End in 0, 2, 4, 6 or 8
Odd numbers	1, 3, 5, 7, 9	Odd numbers end in 1, 3, 5, 7 or 9
Integers	-9, -3, 4, 8, 27, 368	Integers are whole numbers
Multiples	(of 3) 3, 6, 9, 12, 15, 18, …	The product of a number multiplied by any integer (3 multiplied by any integer would be a multiple of 3)
Factors	(of 12) 1, 2, 3, 4, 6, 12	A factor is a number that divides exactly into a number without a remainder
Prime numbers	2, 3, 5, 7, 11, 13, …	Have exactly two different factors. e.g. 7 can only be made by 1 x 7
Square numbers	1, 4, 9, 16, 25, …	A square number results when a number is multiplied by itself e.g. $5^2 = 5 \times 5 = 25$. So 25 is a square number.
Cube numbers	1, 8, 27, 64, 125, …	A cube number results when a number is multiplied by itself three times e.g. $4^3 = 4 \times 4 \times 4 = 64$. So 64 is a cube number.
Triangular numbers	1, 3, 6, 10, 15, …	Starting at 1, each new number is the difference between the two previous numbers add 1.

Fraction, Decimal and Percentage Equivalence

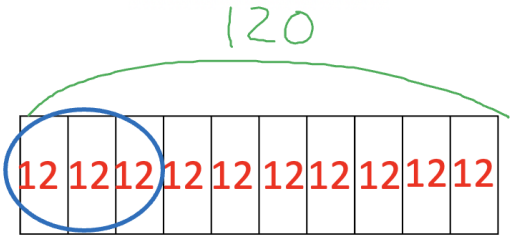


Fractions

Equivalence/Simplifying

Equivalence	Simplifying
  Equivalent fractions are all in proportion to one another. They are formed when the numerator and denominator have the same multiplier.	 To simplify, find a common factor, preferably the highest common factor, and divide the numerator and denominator by that same number. You might need to do this multiple times if you didn't use the highest common factor.

Fraction of an Amount

Method 1	Method 2
$\frac{3}{10}$ of 120  120 ÷ 10 = 12 12 x 3 = 36 Use of a bar model.	$\frac{3}{10}$ of 120 120 ÷ 10 = 12 (÷ denominator) 12 x 3 = 36 (x numerator)

Adding/Subtracting Proper Fractions

Method	
1) List multiples of 4 and 5 to find the lowest common multiple which is 20. 2) Use equivalent fractions to turn each fraction into a fraction with a denominator of 20. 5 x 4 = 20 so 4 x 4 = 16 4 x 5 = 20 so 3 x 5 = 15 3) Add/subtract the fractions. 4) Turn into a mixed number fraction.	4, 8, 12, 16, 20 5, 10, 15, 20 $\frac{4}{5} + \frac{3}{4}$ $\times\frac{4}{4}$ $\times\frac{5}{5}$ $= \frac{16}{20} + \frac{15}{20}$ $= \frac{31}{20}$ $= 1\frac{11}{20}$

Adding Mixed Number Fractions

Method 1	Method 2
$5\frac{1}{3} + 3\frac{4}{5}$ 5 + 3 = 8 $\frac{1}{3} + \frac{4}{5}$ $\times 5$ $\times 5$ $\times 3$ $\times 3$ $= \frac{5}{15} + \frac{12}{15}$ $= \frac{17}{15}$ $= 1\frac{2}{15}$ $1\frac{2}{15} + 8 = 9\frac{2}{15}$ Partition into adding the whole numbers and then adding the fractions.	$5\frac{1}{3} + 3\frac{4}{5}$ $= \frac{16}{3} + \frac{19}{5}$ $\times 5$ $\times 5$ $\times 3$ $\times 3$ $= \frac{80}{15} + \frac{57}{15}$ $= \frac{137}{15}$ $= 9\frac{2}{15}$ Turn each fraction into an improper fraction first.

Subtracting Mixed Number Fractions

Method 1	Method 2
$4\frac{1}{3} - 3\frac{4}{9}$ $4\frac{1}{3} - 3$ $= 1\frac{1}{3} - \frac{4}{9}$ $= \frac{4}{3} - \frac{4}{9}$ (×3 / ×3 and ×1 / ×1) $= \frac{12}{9} - \frac{4}{9}$ $= \frac{8}{9}$	$4\frac{1}{3} - 3\frac{4}{9}$ $= \frac{13}{3} - \frac{31}{9}$ (×3 / ×3 and ×1 / ×1) $= \frac{39}{9} - \frac{31}{9}$ $= \frac{8}{9}$
Partition the second fraction into 3 and 4/9. Subtract the 3 from the first fraction to cancel it down to 1 and 1/3. Turn the fraction improper and then subtract the remaining 4/9.	Turn each fraction into an improper fraction first.

Multiplying Proper Fractions

Proper x proper	Proper x whole number
$\frac{1}{2} \times \frac{3}{4}$ $= \frac{1 \times 3}{2 \times 4}$ $= \frac{3}{8}$	$\frac{1}{5} \times 3$ $= \frac{1}{5} \times \frac{3}{1}$ $= \frac{1 \times 3}{5 \times 1}$ $= \frac{3}{5}$
1) Multiply the two numerators 1 x 3 2) Multiply the two denominators 2 x 4 3) This gives the fraction 3/8. Simplify if necessary.	When multiplying a fraction by a whole, turn the whole number into a fraction by writing /1.

Multiplying Mixed Number Fractions

Method
$4\frac{3}{5} \times 1\frac{2}{3}$ $= \frac{23}{5} \times \frac{5}{3}$ $= \frac{23 \times 5}{5 \times 3}$ $= \frac{115}{15}$ $= \frac{23}{3}$ $= 7\frac{2}{3}$ Turn the mixed number fractions into improper fractions and then multiply. Write the final answer as a mixed number fraction.

Dividing Proper Fractions

Method 1 (Reciprocal)	
$\frac{3}{5} \div \frac{5}{8}$ $= \frac{3}{5} \times \frac{8}{5}$ Multiply by reciprocal $= \frac{3 \times 8}{5 \times 5}$ $= \frac{24}{25}$	1) Swap the numerator and denominator of the second fraction to create the reciprocal of 5/8 which is 8/5. 2) Multiply the numerators 3 x 8 3) Multiply the denominators 5 x 5 4) This gives the fraction 24/25. Simplify if necessary.

Method 2 (Common denominator)

$\frac{4}{5} \div \frac{3}{4}$

$= \frac{16}{20} \div \frac{15}{20}$ (multiply $\frac{4}{5}$ by $\frac{4}{4}$ and $\frac{3}{4}$ by $\frac{5}{5}$)

$= \frac{16}{15}$ ← Num 1 ÷ Num 2

$= 1\frac{1}{15}$

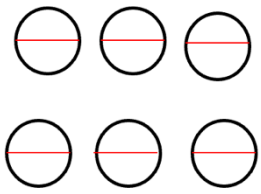
4, 8, 12, 16, 20
5, 10, 15, 20

1) List multiples of 4 and 5 to find the lowest common multiple which is 20.
2) Use equivalent fractions to turn each fraction into a fraction with a denominator of 20.
5 x 4 = 20 so 4 x 4 = 16
4 x 5 = 20 so 3 x 5 = 15
3) Numerator 1 should be divided by numerator 2
4) Turn into a mixed Number fraction.

Dividing Fractions with Wholes.

Method 1

$6 \div \frac{1}{2}$



$= 12$

Demonstrate pictorially using a diagram.

Method 2

$6 \div \frac{1}{2}$

$= \frac{6}{1} \div \frac{1}{2}$

$= \frac{6}{1} \times \frac{2}{1}$ ← Multiply by reciprocal

$= \frac{6 \times 2}{1 \times 1}$

$= \frac{12}{1}$

$= 12$

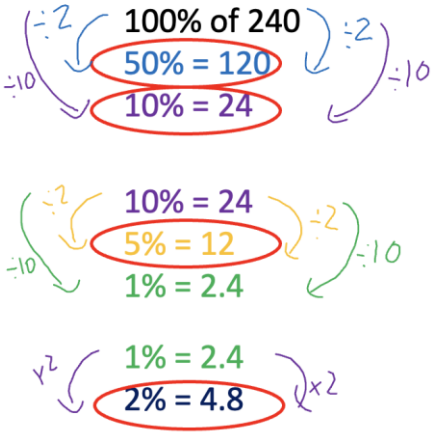
Turn whole into a fraction by writing /1.

Dividing Mixed Number Fractions

Method (Reciprocal)
$3\frac{3}{4} \div 1\frac{2}{5}$ $= \frac{15}{4} \div \frac{7}{5}$ $= \frac{15}{4} \times \frac{5}{7}$ Multiply by reciprocal $= \frac{15 \times 5}{4 \times 7}$ $= \frac{75}{28}$ $= 2\frac{19}{28}$ Turn the mixed number fractions into improper fractions first. Write the final answer as a mixed number fraction.

Percentages

Percentage of Amount: Non Calculator

Method 1	Method 2
Find 67% of 240  120.0 24.0 12.0 4.8 160.8	Convert the percentage to a fraction.

Percentage of Amount: Calculator

Method 1	Method 2
Find 67% of 240 0.67 x 240 = 160.8 Convert the percentage to a decimal multiplier.	Find 67% of 240 67% x 240 = 160.8 Use the percentage button on the calculator knowing 'of' means 'multiply'

Percentage Increase

Method 1	Method 2	Method 3
Increase 240 by 25% 100% + 25% = 125% 1.25 x 240 = 300 Decimal multiplier	Increase 240 by 25% 100% + 25% = 125% 125% x 240 = 300 % button on calculator	Increase 240 by 25% 25% x 240 = 60 240 + 60 = 300 Find 25% and add onto the original amount.

Percentage Decrease

Method 1	Method 2	Method 3
Decrease 240 by 25% 100% - 25% = 75% 0.75 x 240 = 180 Decimal multiplier	Decrease 240 by 25% 100% - 25% = 75% 75% x 240 = 180 % button on calculator	Decrease 240 by 25% 25% x 240 = 60 240 - 60 = 180 Find 25% and subtract from the original amount.

Compound Increase

Method 1	Method 2
£4000 is put in the bank for 5 years. It earns interest of 4% each year. Calculate the total amount at the end of the 5 years. 100% + 4% = 104% $4000 \times 1.04^5 = £4866.61$	£4000 is put in the bank for 5 years. It earns interest of 4% each year. Calculate the total amount at the end of the 5 years. 100% + 4% = 104% $4000 \times 104\%^5 = £4866.61$

Compound Decrease

Method 1	Method 2
A car was bought for 20000 and depreciated in value by 20% each year. Calculate its value after 4 years. 100% - 20% = 80% $20000 \times 0.8^4 = £8192$	A car was bought for 20000 and depreciated in value by 20% each year. Calculate its value after 4 years. 100% - 20% = 80% $20000 \times 80\%^4 = £8192$

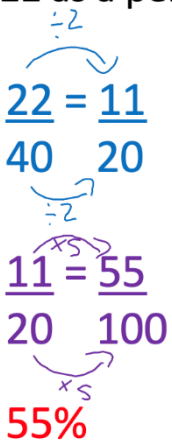
Percentage Change

Method	
A car was bought for £20000 and sold for £15000. Work out the percentage loss. £20000 - £15000 = £5000 $\frac{\text{Change}}{\text{Original}}$ $\frac{5000}{20000} \times 100 = 25\%$	1) Find the difference between the original and new amounts. 2) Divide the change by the original amount and then multiply by 100

Reverse Percentages

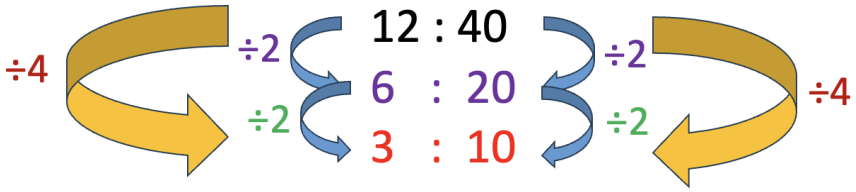
Method	
A computer had 20% off in a sale. The sale price was £320. What was the original price? 100% - 20% = 80% ÷8 ↓ 80% = £320 ÷8 ↓ ×10 ↓ 10% = £40 ↓ ×10 100% = £400	1) Work out what percentage the amount represents. 2) Divide to find a suitable percentage that can be multiplied to make 100 e.g. 8 for 10%, 4 for 20%, 80 for 1% 3) Multiply to turn it into 100%, giving the original amount

One Number as a Percentage of Another

Non-Calculator	Calculator
Write 22 as a percentage of 40  Write as a fraction and then use equivalent fractions to turn into a fraction out of 100.	Write 22 as a percentage of 40 $\frac{22}{40} \times 100 = 55\%$ Type in as a fraction and then multiply by 100.

Ratio and Proportion

Simplifying Ratios

Method
   To simplify, find a common factor, preferably the highest common factor, and divide all parts of the ratio by that same number. You might need to do this multiple times if you didn't use the highest common factor.

Dividing into a Ratio

Method 1

Share £80 in the ratio 3:2.

3 | 16 | 16 | 16 | = 48

2 | 16 | 16 | = 32

80 ÷ 5 = 16

48:32

1) Draw out the bar models
2) Divide 80 into 5 equal parts
   80 ÷5 = 16
3) Write 16 in each part as all parts are equal.
4) Total up 3 parts which is worth 48 and 2 parts which is worth 32

Method 2

Share £80 in the ratio 3:2.

		Total
3 (x16) 48	2 (x16) 32	5 (x16) 80

48:32

1) Set out as a table.
2) Find the total of the parts 3 + 2 = 5
3) Find the multiplier from the total number of parts to the total
   80 ÷5 = 16
1) Multiply each of the parts of the ratio by 16.

Dividing into a Ratio (Given One Value)

Method 1

Paul and Steve share some money in the ratio 3:5. Paul receives £60. How much does Steve get?

Paul | 20 | 20 | 20 | = £60

Steve | 20 | 20 | 20 | 20 | 20 | = £100

60 ÷ 3 = 20

1) Draw out the bar models
2) Write the total of £60 for Paul's three parts so 60 ÷ 3 = 20
3) Write 20 in each remaining part as all parts are equal.
4) Total up 5 parts which is worth £100

Method 2

Paul and Steve share some money in the ratio 3:5. Paul receives £60. How much does Steve get?

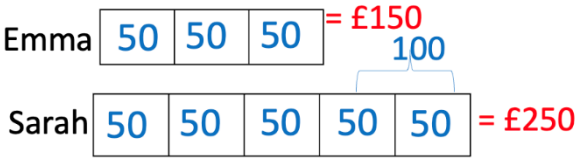
Paul	Steve	Total
3 (x20) £60	5 (x20) £100	

1) Set out as a table.
2) Write £60 underneath Paul's 3 parts
3) Find the multiplier from the three parts to the total
   60 ÷ 3 = 20
4) Multiply the 5 parts by 20 to get £100.

Dividing into a Ratio (Difference)

Method 1

Emma and Sarah share some money in the ratio 3:5. Sarah receives £100 more than Emma. How much money does Emma get?



100 ÷ 2 = 50

Emma gets £150

1) Draw out the bar models
2) Show that 2 parts is worth £100
£100÷ 2 = 50
3) Write 50 in each remaining part as all parts are equal.
4) Total up Emma's and Sarah's parts.

Method 2

Emma and Sarah share some money in the ratio 3:5. Sarah receives £100 more than Emma. How much money does Emma get?

Emma	Sarah	Total
3 ×50 £150	5 ×50 £250	

5 – 3 = 2 Emma gets £150

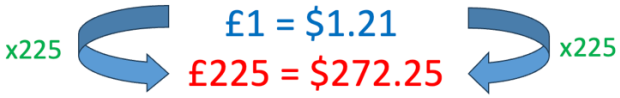
100 ÷ 2 = 50

1) Set out as a table.
2) Find the difference in parts is 2 so
100 ÷ 2 = 50
3) Multiply the 3 and 5 parts by 50.

Direct Proportion

Example 1

The exchange rate is £1 = $1.21. Convert £225 into to $.



Direct proportion occurs when the ratio between two quantities is constant. Both amounts will have the same multiplier and this can be used to find missing quantities.

Example 2

A map has a scale of 1:40000
On the map, the distance between two houses is 9cm.
What is the actual distance between them in metres?

100cm = 1 metre

360000 ÷ 100 = 3600m

Standard Form

Conversions

Place value grids are used to multiply by positive and negative powers of ten. Standard from should be written as a number between 1 and less than 10 multiplied by a power of ten.

Ten Millions	Millions	Hundred Thousandths	Ten Thousandths	Thousands	Hundreds	Tens	Ones	Tenths	Hundredths	Thousandths
							1	8	7	4
	1	8	7	4	0	0	0			

Write 1.874×10^6 as an ordinary number.

1,874,000

Ten Millions	Millions	Hundred Thousandths	Ten Thousandths	Thousands	Hundreds	Tens	Ones	Tenths	Hundredths	Thousandths
				8	7	6	5			
							8	7	6	5

Write 8765 in standard form.

8.765×10^3

Ten Millions	Millions	Hundred Thousandths	Ten Thousandths	Thousands	Hundreds	Tens	Ones	Tenths	Hundredths	Thousandths
							6	7		
							0	0	6	7

Write 6.7×10^{-2} as an ordinary number.

0.067

Ten Millions	Millions	Hundred Thousandths	Ten Thousandths	Thousands	Hundreds	Tens	Ones	Tenths	Hundredths	Thousandths
							0	0	3	5
					0	0	3	5		

Write 0.035 in standard form..

3.5×10^{-2}

Multiplying and Dividing in Standard Form

Multiplying Method 1	Multiplying Method 2
$(4 \times 10^5) \times (2 \times 10^4)$ $4 \times 2 \times 10^5 \times 10^4$ $4 \times 2 = 8$ $10^5 \times 10^4 = 10^9$ 8×10^9 Multiplication is commutative so we can multiply the numbers in any order. 1) Multiply non-powers of 10 2) Multiply powers of ten using index laws. 3) Put the two answers back together.	$(4 \times 10^5) \times (2 \times 10^4)$ 400000×20000 8000000000 8×10^9 1) Turn each of the numbers in standard form into ordinary numbers. 2) Multiply the ordinary numbers. 3) Write the answer in standard form.

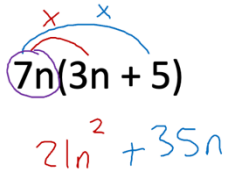
Dividing Method 1	Dividing Method 2
$(8 \times 10^5) \div (2 \times 10^3)$ $8 \div 2 = 4$ $10^5 \div 10^3 = 10^2$ 4×10^2 1) Divide the numbers. 2) Divide the powers of 10 using index laws.	$(8 \times 10^5) \div (2 \times 10^3)$ $80000 \div 2000$ $\frac{80000}{2000} = 40$ 4×10^2 Divide using ordinary numbers.

Adding and Subtracting in Standard Form

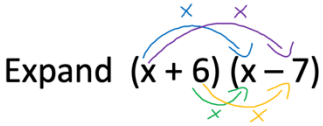
Adding	Subtracting
$5.2 \times 10^4 + 3.6 \times 10^3$ 52000 + 3600 55600 5.56×10^4 1) Turn the numbers into ordinary numbers and carry out column addition. 2) Turn back into standard form.	$5.6 \times 10^4 - 4 \times 10^3$ 56000 - 4000 52000 5.2×10^4 1) Turn the numbers into ordinary numbers and carry out column subtraction. 2) Turn back into standard form.

Algebra

Expanding Single Bracket

Method 1	Method 2
Expand $7n(3n + 5)$ x | 3n | + 5 7n | $21n^2$ | + 35n $21n^2 + 35n$ Use of grid method to separate out the terms then multiply each term by 7n.	Expand $7n(3n + 5)$  Use of looping method (or Santa's Hat) to multiply 7n by each term.

Expanding Binomials

Method 1	Method 2
Expand $(x + 6)(x - 7)$ x | +6 x | x^2 | +6x -7 | -7x | -42 $x^2 + 6x - 7x - 42$ $x^2 - x - 42$ Use of grid method to separate out terms. Ensure terms from each pair of brackets is kept together. Collect the like terms to finish.	Expand $(x + 6)(x - 7)$  $x^2 - 7x + 6x - 42$ $x^2 - x - 42$ $x(x - 7) + 6(x - 7)$ $x^2 - 7x + 6x - 42$ $x^2 - x - 42$ Use of looping method. Can also be separated out as the expansion of two single brackets.

Either method can also be used for expanding triple brackets.

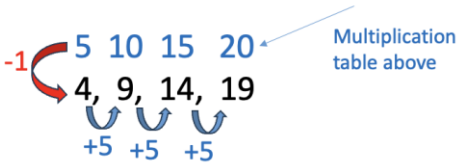
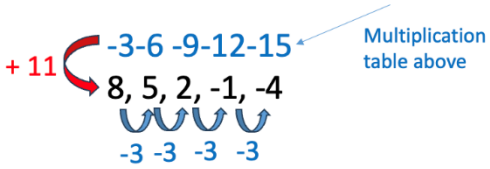
Factorising Single Bracket

Example 1	Example 2
Factorise 12t + 30 1 x 12 2 x 6 3 x 4 t 1 x 30 2 x 15 3 x 10 5 x 6 $6(2t+5)$ 1) List the factor pairs and include any variables 2) Find HCF and circle it and its partner, checking for variable pairs. 3) Put HCF outside bracket and the terms inside the bracket are what is left inside the circle as well as any unused variables.	Factorise $25n^2 - 15np$ 1 x 25 5 x 5 n x n 1 x 15 3 x 5 n x p $5n(5n-3p)$

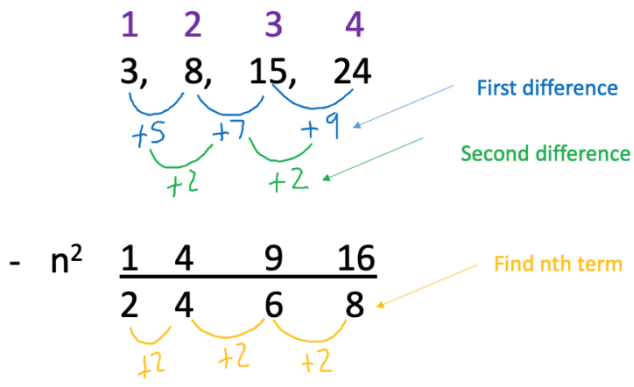
Factorising Quadratics

Method 1	Method 2
Factorise $x^2 + 4x - 32$ Add to make +4 32 1 x -32 2 x -16 4 x -8 -1 x 32 -2 x 16 -4 x 8 Factor pairs for -32 (x - 4) (x + 8) Easier quadratics 1) x^2 means the first term in each bracket must be x 2) Find the factor pairs of -32 3) Identify which factors pairs will add to make +4 4) It's +8 and -4 so these are the missing terms in the brackets	Factorise $2x^2 + 9x + 10$ Add to make +9 2 x 10= 20 1 x 20 2 x 10 4 x 5 $2x^2 + 4x + 5x + 10$ $2x^2 + 4x + 5x + 10$ $2x(x + 2) + 5(x + 2)$ ← Factorise (x + 2) (2x + 5) Trickier quadratics – splitting the middle term method 1) Multiply coefficient of x^2 by end term so 2 x 10 = 20 2) Find factor pairs of 20 3) Identify the pair that adds to make coefficient of x which is 9 (4 and 5) 4) Split middle term 9x into 4x + 5x 5) Factorise first half $2x^2 + 4x$ Factorise second half 5x + 10 6) x + 2 is in both so that's first bracket and left is 2x + 5 making second bracket.

Nth Term Linear Sequences

Ascending	Descending
Find the nth term.  5n - 1 5n - 1 1) Find the common difference -> +5 and this is the coefficient of n so 5n 2) Write the multiplication table for the coefficient above the sequence so 5x 3) Work out the calculation from the 5x table to the sequence -> -1 4) So the nth term is 5n - 1	Find the nth term.  -3n + 11 -3n + 11

Nth Term Quadratic Sequences

Method	
Find the nth term.  $n^2 + 2n$	1) Find the first difference which won't be linear 2) Find the second difference which is linear and a difference of +2 means n^2. First part of nth term is n^2. 3) Write down n^2 sequence and subtract from the original sequence 4) This leaves a sequence of 2, 4, 6, 8. Find the nth term of this sequence which is 2n. 5) So nth term of quadratic sequence is $n^2 + 2n$

Solving Equations

Unknown One Side	Unknowns both sides
$6c - 12 = 48$ $+12 \quad +12$ $\frac{6c}{6} = \frac{60}{6}$ $c = 10$ 1) To solve equations, inverse operations are needed 2) The inverse of -12 is +12 so add 12 to both sides. Any operations done to one side must also be done to the other to maintain the balance. 3) The inverse of x6 is ÷ 6 so c=10 4) Keep all terms lined up throughout.	$9x - 10 = 7x + 24$ $-7x \quad -7x$ $2x - 10 = 24$ $+10 \quad +10$ $\frac{2x}{2} = \frac{34}{2}$ $x = 17$ Extra step required to cancel out the lowest x value first.

Solving Simultaneous Equations

Method

Solve simultaneously

① $4x + 3y = 26$
② $2x + y = 12$ ② × 3
③ $6x + 3y = 36$
− ① $4x + 3y = 26$

$\frac{2x}{2} = \frac{10}{2}$
$x = 5$

Sub x = 5 into ②

$2x + y = 12$
$10 + y = 12$
$-10 \quad -10$
$y = 2$

$x = 5$
$y = 2$

1) Find a common coefficient for either the x or y values. This might involve creating new equations. Ensure all terms are multiplied.
2) Add the equations if the operations are different and subtract if they are the same. This will cancel out either x or y terms.
3) Solve as equation. (x = 5)
4) Substitute known x or y value into one of the original equations and solve (y = 2)

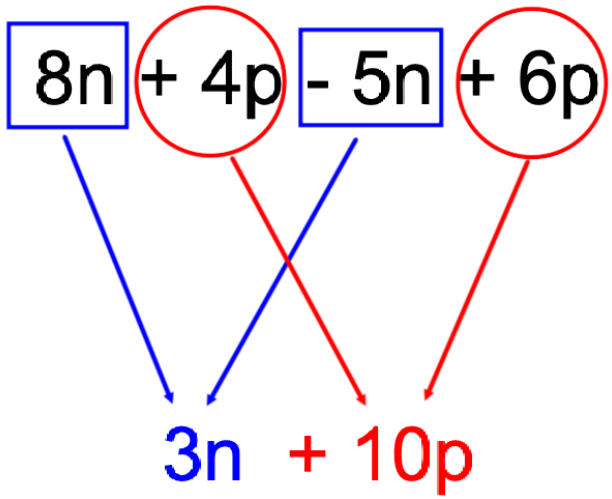
Inequalities

Inequality	Vocabulary
>	Greater than
<	Less than
≥	Greater than or equal to
≤	Less than or equal to

Substitution

Example 1	Example 2
If a = 5 and b = 6, find the value of 7a + 3b. = 7 x 5 + 3 x 6 = 35 + 18 = 53 To substitute, replace the variable with its numerical value and work out the value of the expression.	Formula to convert between Celsius (C) and Fahrenheit (F). F = 1.8C + 32 Convert 2°C into Fahrenheit F = 1.8 x 2 + 32 F = 3.6 + 32 F = 35.6 Substitution shown used in the context of a formula.

Collecting Like Terms

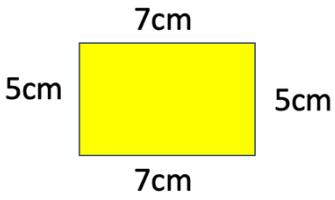
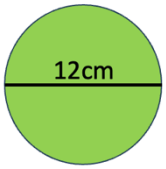
Method
 1) Boxing up method is used to identify like terms. Unlike terms are identified by use of different shapes.

Changing the Subject

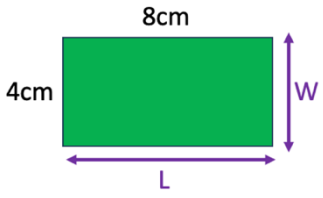
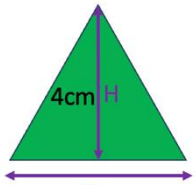
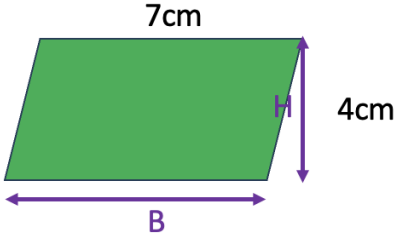
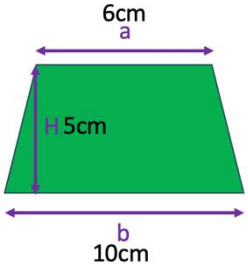
Method	
Make d the subject. $dk - m = t$ $+m \quad +m$ $dk = m + t$ $\div k \quad \div k$ $d = \frac{m + t}{k}$	Steps to change the subject are the same as those to solve an equation through use of inverse operations. The variable which will be made the subject should be isolated on one side. 1) Add m to both sides. 2) Divide both sides by k. 3) The variable d is now isolated on one side making it the subject.

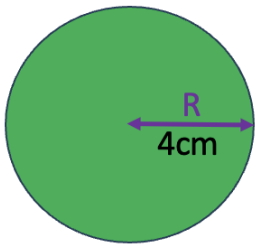
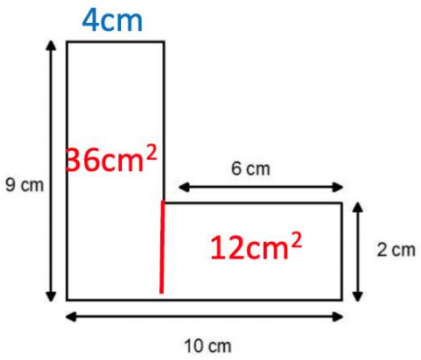
Geometry

Perimeter

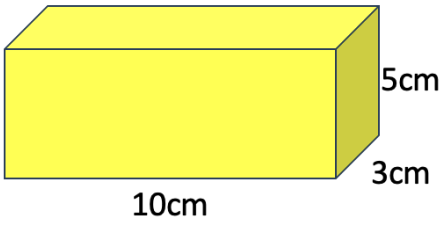
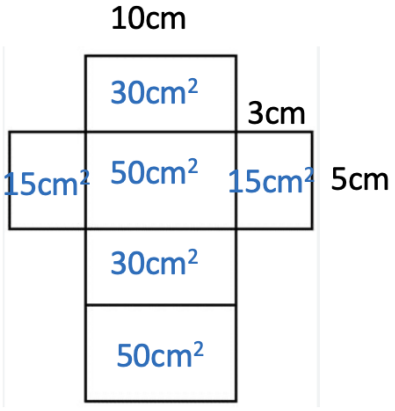
Example 1	Example 2
Work out the perimeter.  P = 7 + 7 + 5 + 5 = 24cm Perimeter is the distance around the outside of a shape and is found by adding all of the outside lengths together.	Calculate the circumference.  C = π x *diameter* C = π x 12 C = 37.7cm (1d.p.) or 12 π The perimeter is a circle is called the circumference. It is found by the formula: circumference = π x diameter. π (pi) is the number 3.14… The circumference can be written as a decimal answer or in terms of π.

Area

Rectangle/Square	Triangle
 Area = length x width A = 8 x 4 A = 32cm^2 Area is the space inside a 2-d shape.	 Area = ½ base x height or $\frac{\text{base x height}}{2}$ A = 4 x 4 A = 16cm^2
Parallelogram	**Trapezium**
 Area = base x height A = 7 x 4 A = 28cm^2	 Area = ½ x (a + b) x height or $\frac{a + b}{2}$ x height A = ½ x (10 + 6) x 5 A = ½ x 16 x 5 A = 40cm^2

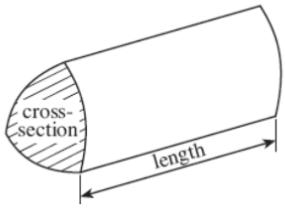
Circle	Compound
 Area = π x radius^2 A = π x 4^2 A = π x 16 A = 50.3cm^2	 Split into 2 rectangles and find missing lengths A1 6 x 2 = 12cm^2 A2 9 x 4 = 36cm^2 A 36 + 12 = 48cm^2

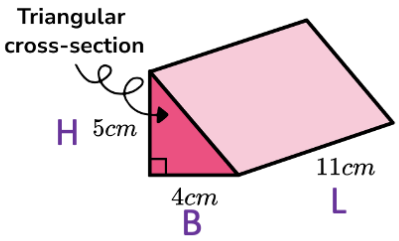
Surface Area

Method
    A1 10 x 3 = 30cm^2 A2 10 x 5 = 50cm^2 A3 3 x 5 = 15cm^2 30 + 50 + 15 = 95cm^2 Double this as there is two of each face. 95 x 2 = 190cm^2 It helps to draw the net and find the area of each face before adding them together.

Volume

Method





Volume = area of the cross-section x length

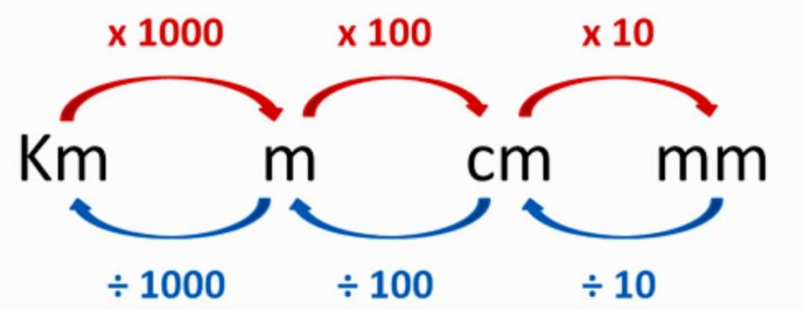
Prisms have a consistent cross-section whilst the other faces are rectangular. Examples include cube, cuboid, triangular prism etc.

V = area of cross section x length
V = ½ B X H X 11
V = 110cm^3

Volume is the space inside a 3-d shape. The same formula applies to all prisms.

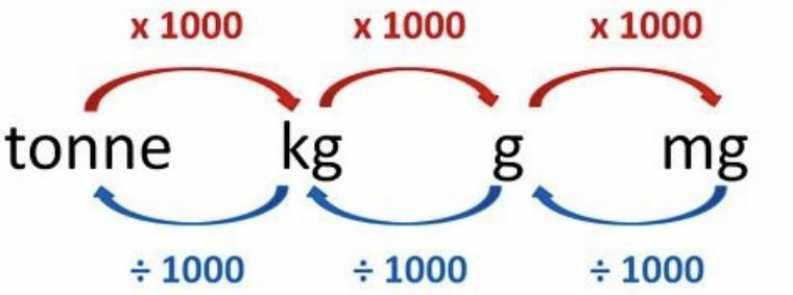
Metric Measures

Length

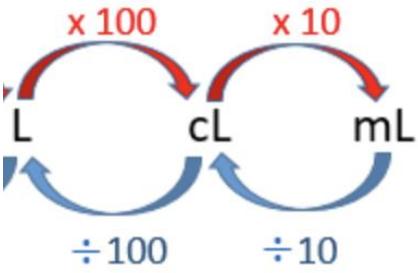


For estimations, a long ruler used in lessons is 30cm, the length of a step is about 1 metre and the height of a door is about 2 metres.

Mass



For estimates, an apple is about 100g and a bag of sugar is 1kg.

Capacity
 For estimates, a can of pop contains 330ml and large bottle of pop 2 litres.

Metric and Imperial Conversions (approximations)

Metric	Imperial
1.6 kilometres	1 mile
2.5 centimetres	1 inch
1 kilogram	2.2 pounds
4.5 litres	1 gallon

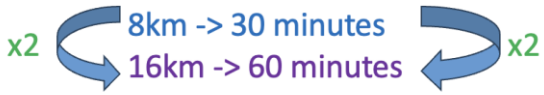
Speed, Distance and Time

$$\text{Speed} = \frac{\text{Distance}}{\text{Time}}$$

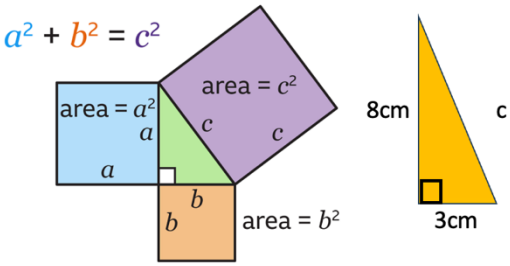
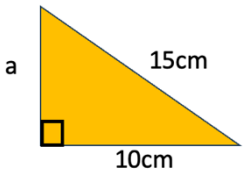
$$\text{Distance} = \text{Speed} \times \text{Time}$$

$$\text{Time} = \frac{\text{Distance}}{\text{Speed}}$$

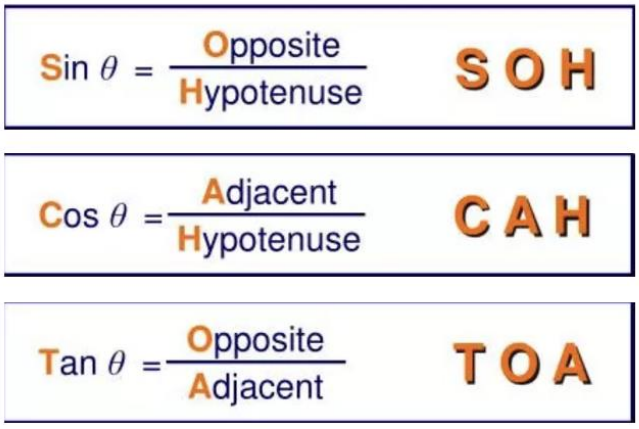
Students will need to know and be able to rearrange the formulas depending on whether they are calculating speed, distance or time.

Speed (Method 1)	Speed (Method 2)
Emma cycles 9 kilometers in 30 minutes. Calculate her average speed in km/h. The  16km/h time of 30 minutes can be scaled up by multiplying by 2 to turn it into 60 minutes.	James runs 15 miles in 2 hours and 30 minutes. Calculate his average speed. 2 hours 30 minutes = 2.5 hours 30 = 0.5 60 15 miles 2.5 hours = 6mph 1) Turn the time into a decimal. This can be done by dividing the minutes by 60 if using a calculator. 2) Substitute the values into the appropriate SDT formula. 3) Give the appropriate unit of measurement.
Distance	Time
A car drives at a speed of 60mph for 4 hours. Calculate the distance travelled. 60mph x 4 hours = 24 miles	A bus travels 160 miles at a speed of 40mph. Calculate how long the journey took. 160 miles = 4 hours 40mph

Pythagoras

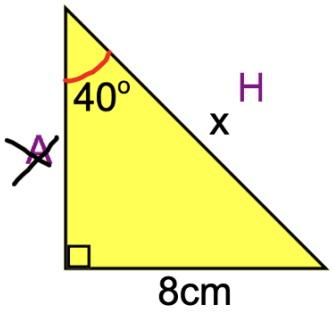
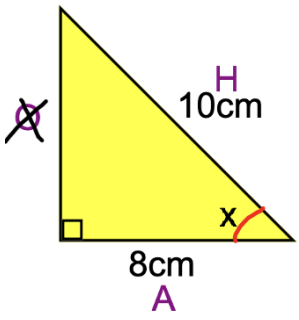
Hypotenuse	Adjacent or opposite
 $a^2 + b^2 = c^2$ $8^2 + 3^2 = c^2$ $64 + 9 = c^2$ $73 = c^2$ $\sqrt{73} = 8.5cm\ (1d.p.)$	 $a^2 + b^2 = c^2$ $a^2 = c^2 - b^2$ $a^2 = 15^2 - 10^2$ $a^2 = 225 + 100$ $a^2 = 325$ $18.0cm\ (1d.p.) = \sqrt{325}$
Pythagoras only applies with right-angled triangles. The hypotenuse (c) is the longest side and is always opposite the right angle.	Rearrange to change the subject of the formula when finding one of the shorter sides.

Trigonometry



Family of 4 Links

Opp ÷ hyp = sin θ
Opp ÷ sin θ = hyp
Hyp x sin θ = opp
Sin x hyp = opp

Missing Side	Missing Angle
 O Hyp = opp / sin θ Hyp = 8 / sin (40) Hyp = 12.4cm 1) Label the hypotenuse (opposite right angle and longest side), opposite (opposite the given angle)and adjacent (next to the given angle). Cross off the side not required. 2) Hypotenuse and opposite mean the ratio of sin is needed. 3) Write down the formula and substitute the given values in. 4) Round to a suitable degree of accuracy.	 Cos θ = adj / hyp Cos $^{-1}$ = (8 / 10) Hyp = 36.9° 1) Label the hypotenuse, opposite and adjacent sides. Cross off the side not required. 2) Hypotenuse and adjacent mean the ratio of cos is needed. 3) Write down the formula and substitute the given values in. 4) When working out the angle it is an inverse operation so cos^{-1} required. 5) Round to a suitable degree of accuracy.

Handling Data

Data Types

Primary	Secondary
Data collected by the person carrying out the study. A first hand source.	Data collected by someone else.

Qualitative	Quantitative
Not numerical data. For example, opinions about the quality of food from a questionnaire.	Numerical data that has been counted or measured. For example, height information about students in a school.

Quantitative data can be further subdivided into two categories:

Discrete	Continuous
Data that can only be certain values and is usually in the form of whole numbers. For example, goals in a football match. You cannot score 2.5 goals.	Data that can be any value such as measurements. For example, a person can weigh 35.67 kg.

Collecting Data

Tally Chart

Transport	Tally	Frequency
Walk	𝍸 𝍸 |||	13
Bus	𝍸 ||	7
Car	||||	4
Bike	𝍸	5
Train	|	1

A tally chart is a useful way of organising a set of raw data. Tallies are useful to record as you go along. A frequency table can be used if this is not necessary.

Grouped Frequency Table

Time, t (minutes)	Frequency
$0 < t \leq 10$	8
$10 < t \leq 20$	21
$20 < t \leq 30$	17
$30 < t \leq 40$	5
$40 < t \leq 50$	2

Grouped data is useful when there are lots of different values. Use of inequalities is necessary when using continuous data.

Representing Data

Pictogram

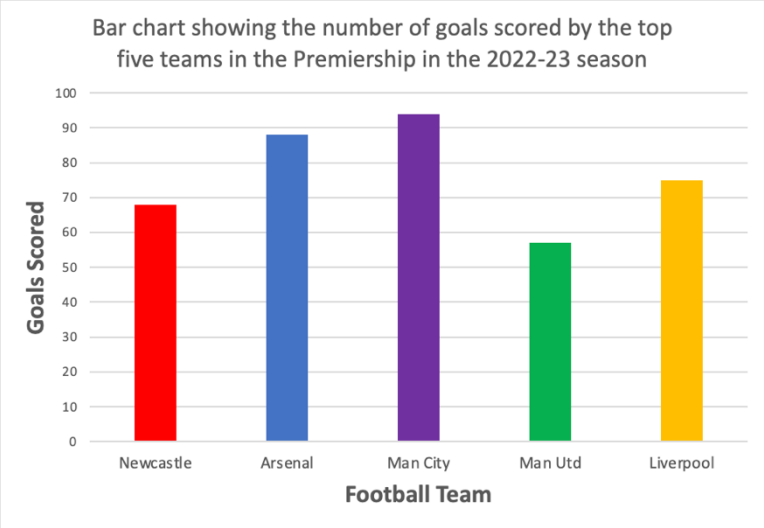
Team	Number of house points
Diamond	★★½
Ruby	★★★
Sapphire	★★★½
Emerald	★★½

Key

★ = 8 points

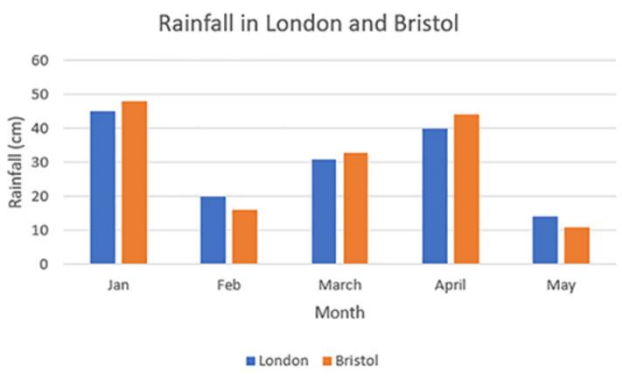
- Pictograms use symbols to represent a frequency
- A key is necessary to identify the quantity that each symbol is worth

Bar Chart



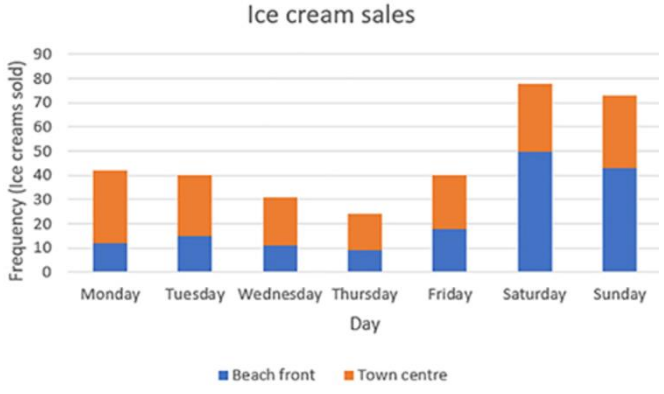
- The x axis is often the categories and the y axis the frequency
- Bars should be equal width and have equal spacing between
- Categories should be labelled under the bars or with a key
- X and y axes should be labelled
- The scale for the frequency should be equally spaced
- There should be spaces between the bars when data in discrete

Dual



Used to compare two sets of data

Composite



Useful for comparing two sets of data that are connected e.g. ice cream sold in two places in the same town.

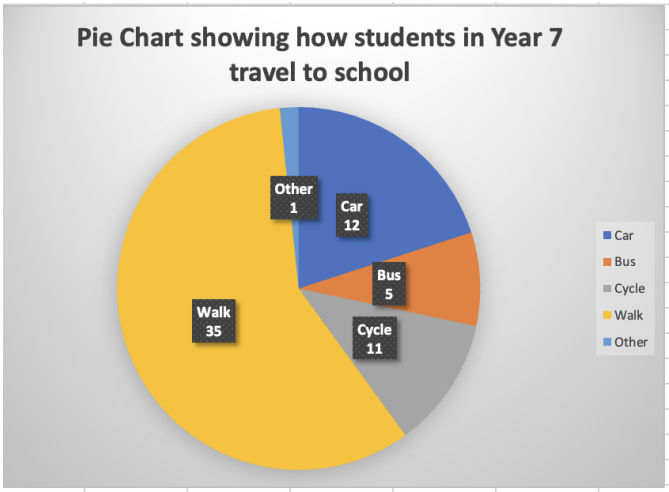
Pie Chart

Working

Method of Travel	Frequency	Calculation
Car	12	$\frac{360}{60} \times 12 = 72^o$
Bus	5	$\frac{360}{60} \times 5 = 30^o$
Cycle	7	$\frac{360}{60} \times 7 = 42^o$
Walk	35	$\frac{360}{60} \times 35 = 210^o$
Other	1	$\frac{360}{60} \times 1 = 6^o$
Total	60	

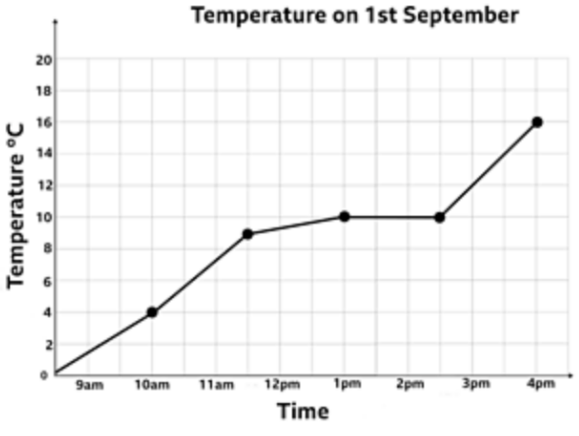
1) There are 360 degrees in a circle so divide this by the total frequency so 360 ÷ 60 = 6° (each person).
2) Multiply each frequency by 6 to get the angle size for each category
3) Check the total of the angles adds up to 360° before drawing the pie chart

Chart



The pie chart should have a key or be appropriately labelled to identify which colour represents which category.

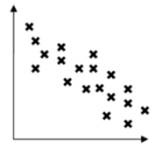
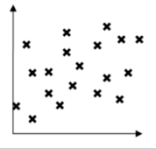
Line Graph

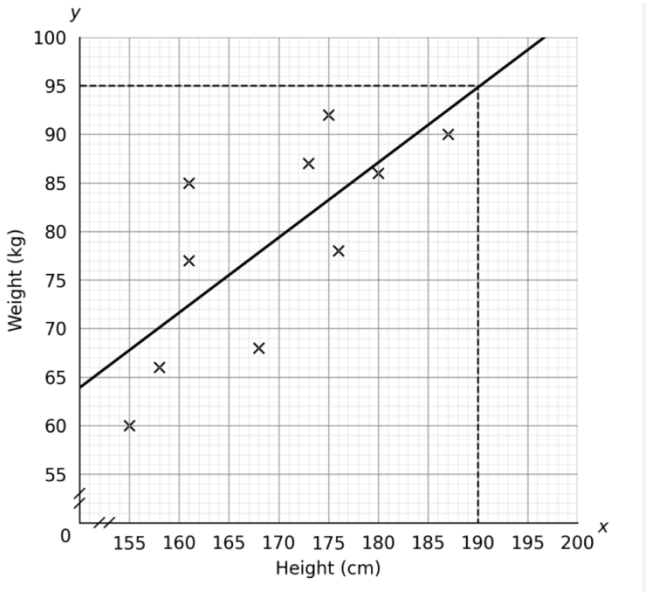


Line graphs are very useful in showing changes over a period of time. The x axis will show time and the y axis the variable being changed or measured (in this case temperature). The data points will be joined with a straight line or smooth curve as appropriate.

Line graphs are useful at showing trends in data and can be used to make predictions.

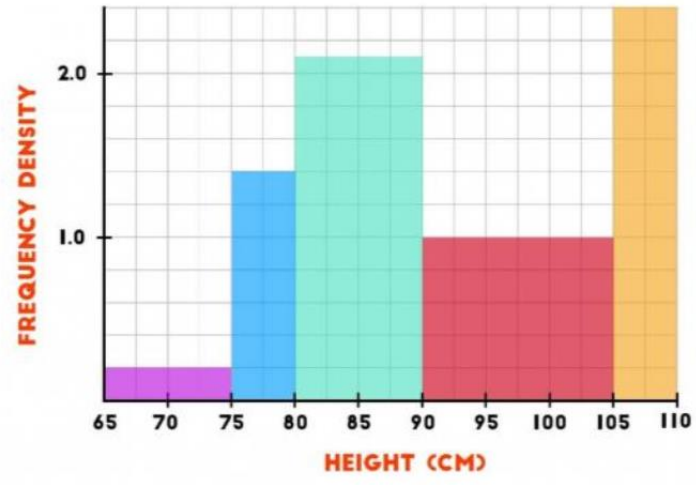
Scatter graphs

Type of correlation	Typical graph	Trend observed
Positive correlation		As one variable increases, the other variable increases.
Negative correlation		As one variable increases, the other variable decreases.
No correlation		There is no obvious relationship between the variables.



Scatter graphs identify whether there is a relationship between two variables. One variable goes on the x axis and the other on the y axis. It does not matter which axis the variables go on. A line of best fit can be drawn when there is a positive or negative correlation. A good line of best fit has approximately the same number of points above and below the line. The line of best fit can be used to approximate the value of one variable when given the other. For example, estimate the height of a child weighing 80kg. The line shows the child would be approximately 170cm.

Histograms



- With histograms the frequency is represented by the width of the bar rather than its height
- Unlike bar charts, histograms do not have spaces between the bars because they are drawn from continuous data
- The y axis should be labelled 'frequency density' and not just 'frequency'

Histograms are commonly confused with bar charts. Unlike bar charts, the bars do not have to be an equal width.

Averages

Mode

Example 1

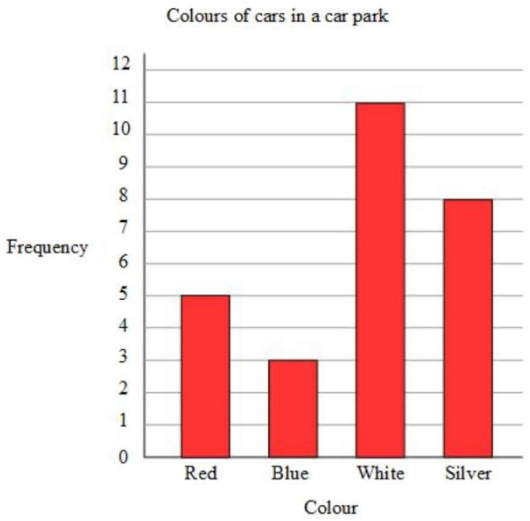
Find the mode.

5, 3, 4, 7, 7, 5, 6, 5, 4

Mode is 5.

The mode is the most common value in a data set and can be non-numerical as well as numerical. For example, most common type of ice-cream flavour. There can be one mode, multiple modes or no mode.

Example 2



The modal colour in the bar graph is white.

Median

Example 1

Find the median.

9, 3, 1, 8, 3, 6

Order from smallest to largest.

1, 3, 3, 6, 8, 9

Halfway is 4.5

(or add together and divide by 2
3 + 6 = 9
9 ÷ 2 = 4.5)

The median is the middle number when ordered form smallest to largest. There can only be one median so if there are two numbers in the middle, the halfway point must be found.

Example 2

Find the median shoe size.

Shoe Size	Frequency	Cumulative Freq	
3	6	6	No
4	8	6 + 8 = 14	No
5	11	6 + 8 + 11 =25	Yes
6	9	6 + 8 + 11 + 9 =34	
7	3	6 + 8 + 11 + 9 + 3 = 37	

37 ÷ 2 = 18.5 -> 19th data point

Median is 5

1) Add the frequencies together and divide by 2. If the answer is odd, round it up to the nearest whole number. **6.5 -> 7th value**
2) If the answer is even, it is that value and the next one up. **17 -> 17th and 18th values**
3) Find the cumulative frequencies.
4) Identify which cumulative frequency the show size is in.

Mean

Example 1

Find the mean.

12, 16, 4, 6, 22, 12

Add together the values.

12 + 16 + 4 + 6 + 22 + 12 = 72

Divide by the number of values.

72 ÷ 6 = 12

Mean is 12

Example 2

Find the estimated mean.

Playing time t (hours)	Frequency	Midpoint	F X M
$0 < t \leq 4$	12	2	12 x 2 = 24
$4 < t \leq 8$	18	6	18 x 6 = 108
$8 < t \leq 12$	19	10	19 x 10 = 190
$12 < t \leq 16$	6	14	6 x 14 = 84
$16 < t \leq 20$	2	18	2 x 18 = 36
	57		442

Add together F X M
24 + 108 + 190 + 84 + 36 = 442
Divide by the frequency total
442 ÷ 57 = 7.8 (1d.p.)
Estimated mean is 7.8

With grouped data the mean is estimated because the actual values are no longer known. The midpoint of the groups is used instead.

Range

Example 1

Find the range.

12, 16, 4, 6, 22, 12

The range is the difference between the highest and lowest values.

22 – 4 = 18

Range is 18

The range is a measure of how spread out the data is.

Example 2

Work out the range.

Lowest

Goals Scored	Frequency
0	5
1	8
2	7
3	4
4	2

Highest

4 – 0 = 4

Range is 4